8.3 Fraunhofer diffraction of many slits

Figure 7 shows a plate with many slits. The slits are very long along the y direction. The period between neighboring slits along z is a and the slit with is b. Since the diffraction pattern should be identical along the Z direction at least near the center, we can choose a point P on a X-Z plane. The electric field E at P with distance Z from the center line, according to Eq, (8.5) is

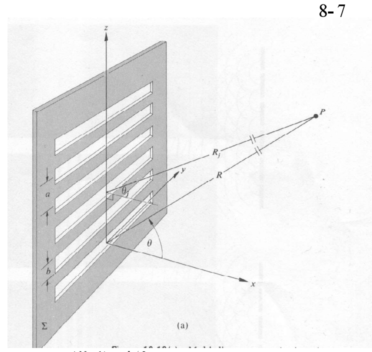

Figure 7 Multi-slit geometry. Point P is far from the diffraction plate.

$$E = \frac{\varepsilon_A e^{i(\omega t - kR)}}{R}\{\int_{-b/2}^{b/2} e^{izkZ/R}dz + \int_{a-b/2}^{a+b/2} e^{izkZ/R}dz + \int_{2a-b/2}^{2a+b/2} e^{izkZ/R}dz + ... + \int_{(N-1)a-b/2}^{(N-1)a+b/2} e^{izkZ/R}dz\}$$

$$= \frac{\varepsilon_A e^{i(\omega t - kR)}}{R}\{\int_{-b/2}^{b/2} e^{izkZ/R}dz + \int_{-b/2}^{b/2} e^{i(z'+a)kZ/R}dz' + \int_{-b/2}^{b/2} e^{i(z'+2a)kZ/R}dz' + ... + \int_{-b/2}^{b/2} e^{i[z'+(N-1)a]kZ/R}dz'\}$$

$$= \frac{\varepsilon_A e^{i(\omega t - kR)}}{R}\int_{-b/2}^{b/2} e^{izkZ/R}dz\,\{1 + e^{iakZ/R} + e^{i2akZ/R} + ... + e^{i(N-1)akZ/R}\}$$

$$= \frac{\varepsilon_A e^{i(\omega t - kR)} b}{R}\frac{\sin\beta}{\beta}\frac{1 - e^{iNakZ/R}}{1 - e^{iakZ/R}} = \frac{\varepsilon_A e^{i[(\omega t - kR) + (N-1)akZ/2R]} b}{R}\frac{\sin\beta}{\beta}\frac{(e^{iNakZ/2R} - e^{-iNakZ/2R})/2i}{(e^{iakZ/2R} - e^{-iakZ/2R})/2i}$$

$$= \frac{\varepsilon_A e^{i[(\omega t - kR) + (N-1)akZ/2R]} b}{R}\frac{\sin\beta}{\beta}\frac{\sin N\alpha}{\sin\alpha} \qquad (8.10)$$

with $\alpha = akZ/2R = (ka/2)\sin(\theta)$ (8.11)

$$I = EE^*/2 = \frac{1}{2}(\frac{\varepsilon_A b}{R})^2(\frac{\sin\beta}{\beta})^2(\frac{\sin N\alpha}{\sin\alpha})^2$$

When $\theta = 0$, $I(0) = \frac{1}{2}(\frac{\varepsilon_A b}{R})^2 \bullet 1 \bullet N^2 = \frac{1}{2}(\frac{\varepsilon_A b}{R})^2 N^2$, so

$$I(\theta) = \frac{I(0)}{N^2}(\frac{\sin\beta}{\beta})^2(\frac{\sin N\alpha}{\sin\alpha})^2 \qquad (8.12)$$

When N=1, $I(\theta) = I(0)(\frac{\sin\beta}{\beta})^2$, which is the single slit diffraction formula of Eq. (8.9).

When N=2, $I(\theta) = I(0)(\frac{\sin\beta}{\beta})^2\cos^2\alpha$, which is double slit diffraction formula. When the slit width is very small (b/ λ<<1 so β <<1), this formula is identical to the intensity formula (7.24) of Young's double slit diffraction.

The **Principal maxima** occur when both $(\sin N\alpha = 0)$ and $(\sin\alpha = 0)$, that is $\alpha = 0, \pm\pi, \pm 2\pi, ...m\pi,$ or equivalently, since $\alpha = (ka/2)\sin\theta$,(with m an integer)

$$a\sin\theta_m = m\lambda \qquad (8.13)$$

There are (N-2) **subsidiary maxima** between two neighboring principal maxima. They occur when $(\sin N\alpha) = \pm 1$.

$$\alpha = \pm\frac{\pi}{2N}, \pm\frac{3\pi}{2N}, \pm\frac{5\pi}{2N}, ... \qquad (8.14)$$

There are (N-1) **Minima** between two principal maxima. They occur when $\sin N\alpha = 0$ *but* $\sin\alpha \neq 0$.

$$\alpha = \pm\frac{\pi}{N}, \pm\frac{2\pi}{N}, \pm\frac{3\pi}{N}, \pm\frac{(N-1)\pi}{N}, \pm\frac{(N+1)\pi}{N}, ... \qquad (8.15)$$

Figure 8 shows the formation of multiple-slit diffraction pattern with a=4b and N=6.

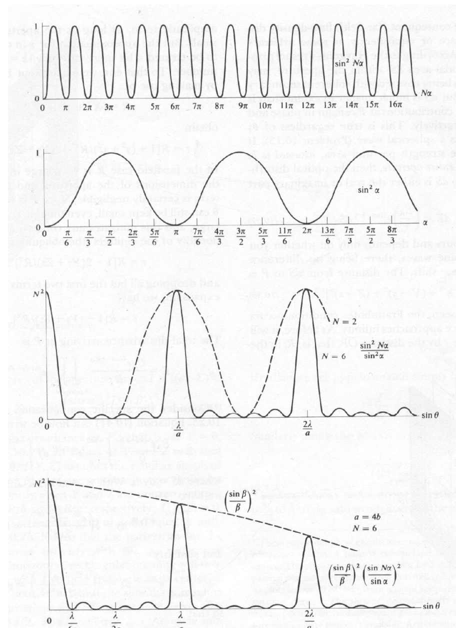

Figure 8 Multiple-slit pattern (a=4b, N=6)

Figure 9 shows diffraction patterns for systems with different number of slits as depicted on the left side of the figure. In all patterns, the two minima on each side of the central maxima are determined by the slit width b. When N >1, several principal maxima can be seen in the central region. When N is getting larger, (N-1) minima and (N-2) subsidiary maxima can been seen between two principal maxima.

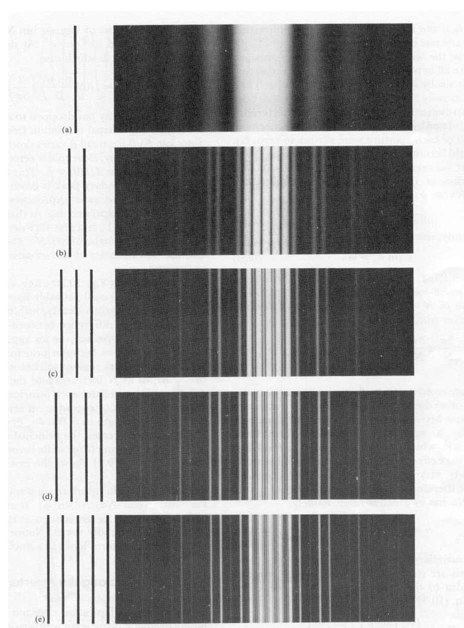

Figure 9 Diffraction patterns for slit systems shown at left

8.3 Diffraction grating

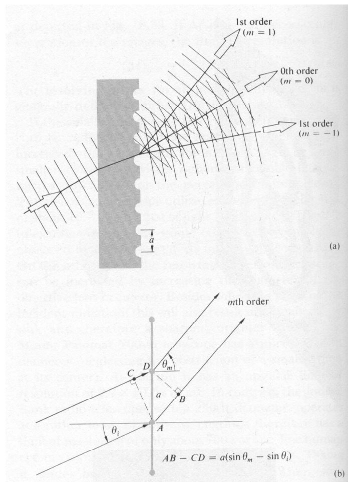

Fig 10 A transmission grating

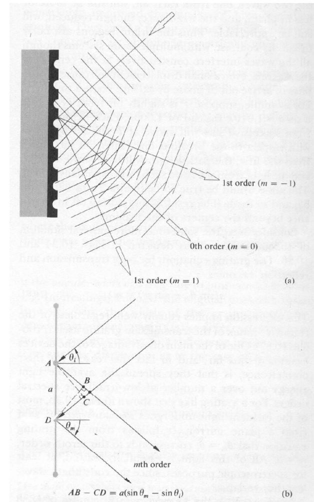

Fig. 11 A reflection grating

A repetitive array of diffracting elements, either apertures or obstacles, that has the effect of producing periodic alternations in the phase , amplitude, or both of an emergent wave is said to be a **diffraction grating**.

One of the simplest such arrangements is the multiple-slit configuration shown in figure 7, which is a **transmission grating**. Another kind of transmission grating is shown in figure 8.10(a), which has periodic parallel notches into the surface of a flat, clear glass plate. Each of the notches serves as a source of scattered light, and together they form a regular array of parallel line sources. You can envision the wavelets as radiated with different phases over the grating surface. An emerging wavefront therefore contains periodic variations in its shape. This in turn is equivalent to an angular distribution of constituent plane waves

On reflection from this kind of grating, light scattered by the various periodic surface features will arrive at some point P with a definite phase relationship. The consequent diffraction pattern generated after reflection is quite similar to that arising from transmission. A grating designed to function in this fashion is called as a **reflection grating** .

When the incident light is normal to the grating surface, the direction for maximum irradiances is governed by Eq. (8.13), known as grating equation:

$$a\sin\theta_m = m\lambda \qquad (8.13)$$

The integer m is called order. The 0[th] order corresponds to a specular beam.

When the incident light makes an angle θ_i with the surface normal, the path length difference between beams scattered by two identical adjacent feature is, according to fig 10 & fig 11,